somewhere in a probability distribution of misfits from all of the possible experiments. From this distribution of misfits we can find the misfit value, $M$, such that a certain percentage of all misfits would have misfits less than $M$. If we know that 95% of all of the misfits from these experiments have misfits $m < M$, then we have found the 95% confidence interval.

It should be stated that we are not considering the distributions of $m$ values calculated in searching for the best fitting model for our single experiment. In our single experiment the best model has an misfit $m$ at one end of a distribution of misfits corresponding to slightly different stress states. Instead, consider that when the best model for our experiment has been identified, all of the other possible stress states for the data in this experiment are ignored. Here we consider the distribution of best fitting models, or misfits, that would be found if we were able to redrill all of the holes along the same well paths and measure the breakout orientations in those new holes.

We will use the results of *Parker and McNutt* [1980], who calculated the statistics of the one-norm misfit,

$$m = \sum_{j=1}^{n} |x_j| / \sigma_j \tag{13}$$

when $x_j$ are independent normal random variables with zero mean and standard deviations $\sigma_j$. The expected value of $m$ is

$$\overline{m} = \sqrt{\frac{2}{\pi}} n \approx 0.79788n. \tag{14}$$

*Parker and McNutt* [1980] also wrote a program, which, given a probability $P$ and $n$, calculates the misfit $M$ such that the probability is $P$ that $m \leq M$. In other words, this program finds $M(P,n)$ such that the integral from $-\infty$ to $M$ of the one-norm distribution is $P$ for a one-norm distribution of the order of $n$. We use $M(P,n)$ later to calculate the 95% confidence limit.

To find the 95% confidence limit, we begin by defining $\sum_{min}$ to be the minimum misfit observed in our search for the best fitting model. We then assume that this particular one-norm misfit corresponds to the mean misfit of a one-norm distribution. Of course, the mean misfit and the minimized misfit will differ, so we assume that we incorrectly estimated the errors, $\sigma_j$, in our misfit calculation. To correct this estimation, we introduce a new constant factor, $f$, by which we multiply all of the standard deviations, $\sigma_j$, such that a new misfit sum is equal to the expected misfit. Since we have already found the best fitting model to our data, we have lost $k$ degrees of freedom, where $k$ is the number of variables in the problem. We then should use the expected one-norm mean for $n-k$ observations. Mathematically, this equation becomes

$$\begin{aligned} \sum_{min} &= \sum_{j=1}^{n} \frac{|\theta_j - o_j|}{\sigma_j} \\ (2/\pi)^{1/2}(n-k) &= \sum_{j=1}^{n} \frac{|\theta_j - o_j|}{f\sigma_j} \qquad \text{from (14)} \\ &= \frac{\sum_{min}}{f} \\ f &= \frac{\sum_{min}}{(2/\pi)^{1/2}(n-k)} \end{aligned}$$

Finally, to find the $P$% confidence misfit value, $\sum_P$, we use the following expression

$$\frac{\sum_P}{f} = M(P, n-k) \tag{15}$$

where $M(P, n-k)$ is the function given by *Parker and McNutt* [1980]. We still must subtract $k$ from $n$ to find $M$, since we lost $k$ degrees of freedom in the search for the best fitting model. Solving for $\sum_P$, we find

$$\sum_P = \frac{M(P, n-k)}{(2/\pi)^{1/2}(n-k)} \sum_{min}. \tag{16}$$

*Gephart and Forsyth* [1984] find a similar expression in their equation (7), except they assumed that for large $n$ one can replace the one-norm inverse cumulative function $M(P,n)$ with values from standard tables of Gaussian statistics. Since we have a code that calculates $M(P,n)$, there was no reason to make this assumption.

### Error Analysis

Our forward modeling approach does not give any error estimates by default, unlike other inversion methods. We determine our error bounds by varying different parameters of the best fitting stress state and evaluating how much variation is required to raise the misfit to the 95% one-norm confidence level. Because of the nature of successively rotating the principal stress directions for each Euler angle, error bounds for the Euler angles are not of much use. To see how well constrained the principal stress directions are, we fix the stress state ratio $\phi$, iterate the borehole azimuth $\delta$ from $0°$ to $360°$, and iterate the borehole deviation $\Omega$ from $0°$ to $90°$ in $1°$ increments. This iteration rotates the **J** axis across the lower hemisphere of a stereographic plot.